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NOTES ON DERIVATION OF EMPIRICAL CHARTS FOR CALCULATION OF ROTOR BLADE AIRLOADS

Department of Aeronautical Engineering

Report No. 458

May, 1959

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Department of aeronautical ~~Mechanical and~~ ~~Aerospace~~ Engineering

Office of Naval Research
Consulting Project

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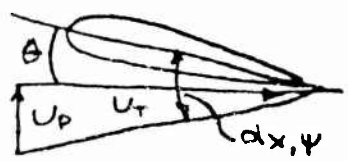
Introduction

The calculation of air load distribution along the rotor blades requires the knowledge of the distribution of local angle of attack.

The local angle of attack is dependent, in addition to the geometric pitch, on the magnitude and direction of local velocity vector, which is dependent on magnitude of free stream velocity, rotor angle of attack, azimuthal position of the blade and the effective induced velocity (induced velocity and air mass inertia effects) and can be expressed in the form

$$\alpha_{x,\psi} = \theta + \tan^{-1}\frac{U_P}{U_T} \qquad (1)$$



The effective induced velocity can be resolved into three mutually perpendicular components. The axial component is predominant along most of the blade. At the tip of the blade the predominant components are axial and radial, however the effect of the lift reduction to zero in that vicinity can be simply expressed in terms of an effective axial induced velocity component. In the regions of flow reversal and low tangential velocities, the magnitude of tangential component of induced velocity is large by comparison with total tangential velocity, but since the airloads in those regions are negligeable and U_P is much larger than U_T, the use of effective axial induced velocity to account for tangential component is justifiable.

The distribution of effective axial induced velocity component can be expressed nondimensionally in the Fourrier form

$$\frac{v(x)}{\Omega R} = \lambda(x) = \lambda_o + \sum \lambda_{na} \cos n\psi + \sum \lambda_{nb} \sin n\psi \qquad (2)$$

The harmonic coefficients vary along the blade and depend on the number of blades b, rotor angle of attack α, advance ratio μ, blade loading $\frac{C_T}{\sigma}$ and on hinged blades on blade mass constant γ. (See equation 7).

The above coefficients can be computed from the experimental pressure distribution data on the rotors with various numbers of blades and various test conditions. Since the coefficients will be plotted in the non-dimension , form, they can be used for calculation of airloads on any rotors. It will be advisable to plot the data first, from the conditions which exclude the effects of abrupt stall and compressibility and treat those phenomena as separate corrections. However, the effect of Mach number on the slope of the blade lift coefficient curves must be accounted in all cases.

It is represented in the following, a suggested method of calculation of the effective axial component of induced velocity harmonic coefficients on the stiff blades, from the pressure distribution measurements. The procedure of computing the airloads on the basis of the plotted coefficients will be also outlined.

The calculation of harmonic coefficients of effective axial induced velocity.

It is quite important that the coefficients computed from the experimental load measurements are reduced to "stiff" blades, in order to make the results applicable for calculation of loads on blades of any stiffness.

Mr. Robert Loewey suggested, as the result of his investigations, that a blade can be classified as a "stiff" one if it has a ratio of first bending frequency to rotating frequency above 4.5. This is quite difficult to achieve considering that a general trend of that ratio is toward 2.5.

The one way of eliminating the effect of flexibility is to install, if it is practical, the accelerometers along the blade and measure the inertia terms, then by integration determine total damping loads. Next find from above, the damping term due to flexibility and subtract the damping due to flexibility from the total airload to obtain the airload on a stiff blade.

That procedure is, theoretically, quite simple, if the coupling between bending and torsion is not strong, but becomes quite hairy if the coupling is appreciable. It was mentioned, originally, to the writer by Mr. G. Brooks from NASA.

The distribution of angle of attack along the blade at any azimuth angle is obtained from the expression for lift.

$$\alpha_{x,\psi} = \frac{dL(x,\psi)}{\frac{1}{2} a_M \rho U_T^2 R\,dx\,C} \tag{3}$$

where

$$U_T = \Omega R(x + \mu \sin\psi)$$

$$a_M = \frac{a_o}{\sqrt{1-M^2}}$$

a_o is a two dimensional slope at low Mach number.

The $\frac{dL}{dx}$ term which is obtained from test data can be espressed in the form

$$\frac{dL}{dx} = \frac{dL_o}{dx} + \sum \frac{dL_{na}}{dx} \cos n\psi + \sum \frac{dL_{nb}}{dx} \sin n\psi \tag{4}$$

The coefficients L_{na} and L_{nb} can be computed from the set of simultaneous equations taking different blade azimuth positions. Measuring airloads every 30 degrees will permit the computation of the lift harmonic coefficients up to the 5th harmonic of ψ.

The following table is a matrix for calculation of harmonic lift coefficients. (P. 5)

The five harmonic coefficients and the constant part $L_0(x)$ at each blade section are obtained by solving eleven simultaneous equations. The constant part $L_0(x)$ is obtained, easily, by averaging the values of "L" at each blade station.

$$L_o(x) = \frac{L(x,0) + L(x,30^\circ) + \cdots + L(x,330)}{12} \qquad (5)$$

As it was mentioned previously

$$\alpha_{x,\psi} = \theta + \tan^{-1}\frac{U_P}{U_T} \cong \theta + \frac{U_P}{U_T} \qquad (1)$$

For flapping rotors with small hinges, the equation for the velocity component U_P is

$$U_P = -xR\dot{\beta} - \mu\Omega R\beta\cos\psi + \mu\Omega R\tan\alpha - v_z \qquad (6)$$

The equation is written in reference to no feathering axis. In the above, α is the rotor angle of attack and is positive, when the free stream velocity V is directed from underneath the rotor.

v_z is the effective axial component of induced velocity and is positive when directed downwards.

Matrix of harmonics for L_{na} and L_{nb} at various values of ψ for azimuth interval of 30°.

	L_{1a}	L_{2a}	L_{3a}	L_{4a}	L_{5a}	L_{6a}	L_{1b}	L_{2b}	L_{3b}	L_{4b}	L_{5b}	L_{6b}
0	1.0	1.0	1.0	1.0	1.0	1.0	0	0	0	0	0	0
30	.866	.5	0	-.5	-.866	-1.0	.5	.866	1.0	.866	.5	0
60	.5	-.5	-1.0	-.5	.5	1.0	.866	.866	0	-.866	-.866	0
90	0	-1.0	0	1.0	0	-1.0	1.0	0	-1.0	0	1.0	0
120	-.5	-.5	1.0	-.5	-.5	1.0	.866	-.866	0	.866	-.866	0
150	-.866	.5	0	-.5	.866	-1.0	.5	-.866	1.0	-.866	.5	0
180	-1.0	1.0	-1.0	1.0	-1.0	1.0	0	0	0	0	0	0
210	-.866	.5	0	-.5	.866	-1.0	-.5	.866	-1.0	.866	-.5	0
240	-.5	-.5	1.0	-.5	-.5	1.0	-.866	.866	0	-.866	.866	0
270	0	-1.0	0	1.0	0	-1.0	-1.0	0	1.0	0	-1.0	0
300	.5	-.5	-1.0	-.5	.5	1.0	-.866	-.866	0	.866	.866	0
330	.866	.5	0	-.5	-.866	-1.0	-.5	-.866	-1.0	-.866	-.5	0

The flapping angle can be expressed as:

$$\beta = \beta_0 - \Sigma a_n \cos n\psi - \Sigma b_n \sin n\psi$$

From equation (1) and (6)

$$v_z = \theta U_T - xR\dot{\beta} - \mu\Omega R\beta\cos\psi + \mu\Omega R\tan\alpha - \alpha_{x,\psi} U_T \qquad (7)$$

We can see from the above equation that v_z must be expressed in terms of trigonometric functions of ψ, and is a function of blade pitch $\theta = f\left(\frac{C_T}{\sigma}\right)$, rotor angle of attack α, advance ratio μ, and, because of β_0, is also a function of blade mass constant γ. It is not apparent from the equation 7, but the distribution of v_z is, from the intuitive reasoning, a function of number of blades, b.

Substituting into equation (7) equations 2 for v_z, 3 and 4 for $\alpha_{x,\psi}$ and multiplying both sides by $x + \mu \sin\psi$, we obtain the following equation for harmonic coefficients of v_z.

$$\begin{aligned}
& x\lambda_0 - \theta\left(x^2 + \frac{\mu^2}{2}\right) - \mu x\tan\alpha + K\frac{dL_0}{dx} \\
& - \left(2\theta x\mu + \mu^2\tan\alpha\right)\sin\psi \\
& + \mu x \beta_0 \cos\psi \\
& + \frac{\mu^2}{2}\beta_0 \sin 2\psi \\
& + \frac{\mu^2}{2}\theta\cos 2\psi \\
& + \sum_{n=1}^{n=\infty}\left(x\lambda_{nb} - nx^2 a_n + K\frac{dL_{nb}}{dx}\right)\sin n\psi \\
& + \sum_{n=1}^{n=\infty}\left(x\lambda_{na} - nx^2 b_n + K\frac{dL_{na}}{dx}\right)\cos n\psi \\
& + \frac{\mu^2}{4}\sum_{n=1}^{n=\infty} a_n \sin(n-2)\psi \\
& - \frac{\mu^2}{4}\sum_{n=1}^{n=\infty} b_n \cos(n-2)\psi
\end{aligned} \qquad (8)$$

$$\sum_{n=1}^{n=\infty}\left(-\frac{\mu}{2}\lambda_{na}-(n+1)\frac{\mu x}{2}b_n\right)\sin(n-1)\psi$$

$$+\sum\left(\frac{\mu}{2}\lambda_{nb}-(n+1)\frac{\mu x}{2}a_n\right)\cos(n-1)\psi$$

$$+\sum\left(\frac{\mu}{2}\lambda_{na}+(n-1)\frac{\mu x}{2}b_n\right)\sin(n+1)\psi$$

$$+\sum\left(-\frac{\mu}{2}\lambda_{nb}+(n-1)\frac{\mu x}{2}a_n\right)\cos(n+1)\psi$$

$$+\sum-\frac{\mu^2}{4}a_n\sin(n+2)\psi$$

$$+\sum\frac{\mu^2}{4}b_n\cos(n+2)\psi$$

$$=0$$

$$\text{where}\quad K=\frac{c}{2}a_M\rho\Omega^2R^3$$

Equating the coefficients of similar trigonometric functions in equation (8) will give us a set of simultaneous equations for harmonic coefficients of induced velocity. If all harmonic coefficients above the 5th are neglected, we have

$$x\lambda_0-\theta\left(x^2+\frac{\mu^2}{2}\right)-\mu x\tan\alpha+K\frac{dL_0}{dx}=0 \tag{9}$$

$$-\left(2\theta x\mu+\mu^2\tan\alpha\right)+\left(x\lambda_{1b}-x^2a_1+K\frac{dL_{1b}}{dx}\right)+\frac{\mu^2}{4}a_3-\left(\frac{\mu}{2}\lambda_{2a}+\frac{3}{2}\mu x b_2\right)=0 \tag{10}$$

$$\mu x \beta_0 + \left(x \lambda_{1a} - x^2 b_1 + \kappa \frac{dL_{1a}}{dx}\right) - \frac{\mu^2}{4} b_3 + \left(\frac{\mu}{2} \lambda_{2b} - \frac{3}{2} \mu x a_3\right) = 0 \tag{11}$$

$$\frac{\mu^2}{2} \beta_0 + \left(x \lambda_{2b} - 2x^2 a_2 + \kappa \frac{dL_{2b}}{dx}\right) + \left(-\frac{\mu}{2} \lambda_{3a} - 2\mu x b_3\right) + \frac{\mu}{2} \lambda_{1a} + + \underline{\frac{\mu^2}{4} a_4} = 0 \tag{12}$$

$$\frac{\mu^2}{2} \theta + \left(x \lambda_{2a} - 2x^2 b_2 + \kappa \frac{dL_{2a}}{dx}\right) + \left(\frac{\mu}{2} \lambda_{3b} - 2\mu x a_3\right) - \frac{\mu}{2} \lambda_{1b} - \underline{\frac{\mu^2}{4} b_4} = 0 \tag{13}$$

$$\left(x \lambda_{3b} - 3x^2 a_3 + \kappa \frac{dL_{3b}}{dx}\right) + + \underline{\frac{\mu^2}{4} a_5} + \left(\underline{-\frac{\mu}{2} \lambda_{4a}} - \underline{\frac{5}{2} \mu x b_4}\right) + \left(\frac{\mu}{2} \lambda_{2a} + \frac{\mu x}{2} b_2\right) - \frac{\mu^2}{4} a_1 = 0 \tag{14}$$

$$\left(x \lambda_{3a} - 3x^2 b_3 + \kappa \frac{dL_{3a}}{dx}\right) - \underline{\frac{\mu^2}{4} b_5} + \left(\underline{-\frac{\mu}{2} \lambda_{4b}} - \underline{\frac{5}{2} \mu x a_4}\right) \left(-\frac{\mu}{2} \lambda_{2b} + \frac{\mu x}{2} a_2\right) + \frac{\mu^2}{4} b_1 = 0 \tag{15}$$

$$\begin{aligned}&\left(x\lambda_{4b} - 4x^2 a_4 + K\frac{dL_{4b}}{dx}\right)\\&\left(-\frac{\mu}{2}\lambda_{5a} - 3\mu x b_5\right) + \left(\frac{\mu}{2}\lambda_{3a} + \mu x b_3\right)\\&- \frac{\mu^2}{4} a_2 = 0\end{aligned} \tag{16}$$

$$\begin{aligned}&\left(x\lambda_{4a} - 4x^2 b_4 + K\frac{dL_{4a}}{dx}\right)\\&\left(\frac{\mu}{2}\lambda_{5b} - 3\mu x a_5\right) + \left(-\frac{\mu}{2}\lambda_{3b} + \mu x a_3\right)\\&+ \frac{\mu^2 b_2}{4} = 0\end{aligned} \tag{17}$$

$$\begin{aligned}&\Big(x\lambda_{5b} - 5x^2 a_5 + K\frac{dL_{5b}}{dx}\\&+ \frac{\mu}{2}\lambda_{4a} + \frac{3}{2}\mu x b_4 - \frac{\mu^2}{4} a_3\Big) = 0\end{aligned} \tag{18}$$

$$\begin{aligned}&\left(x\lambda_{5a} - 5x^2 b_5 + K dL_{5a}\right)\\&- \frac{\mu}{2}\lambda_{4b} + \frac{3}{2}\mu x a_4 + \frac{\mu^2}{4} b_3 = 0\end{aligned} \tag{19}$$

If the solution of induced velocity harmonic coefficients is limited to the third order, only equations 9-15 become valid and the terms underlined in those equations vanish.

Data Presentation

Vertical flight $\mu = 0$

The plot of harmonic coefficients of the effective axial induced velocity component along the blade will be presented varying the following parameters

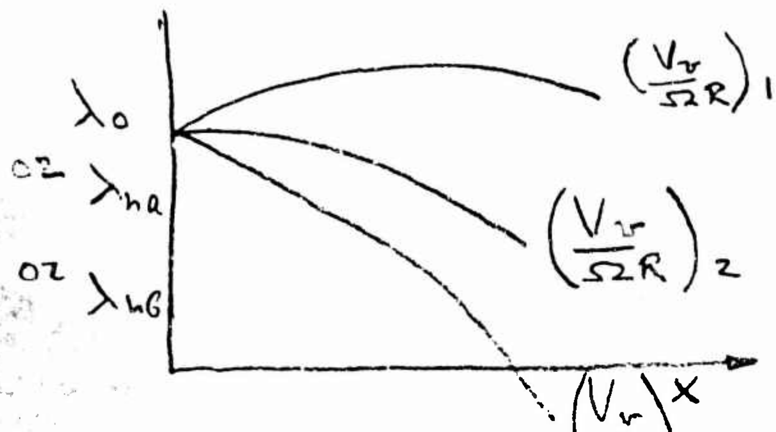
number of blades b =

blade loading $\frac{C_T}{\sigma}$ =

and the ratio $\frac{V_v}{\Omega R} = \frac{\text{vertical velocity}}{\text{tip speed}}$

The variation of $\frac{C_T}{\sigma}$ and $\frac{V_v}{\Omega R}$ is accomplished by varying collective pitch θ and torque.

The charts in vertical flight could be of the form



b =

$\frac{C_T}{\sigma}$ =

Forward flight $\mu \neq 0$.

The charts showing variation of λ_o, λ_{na}, and λ_{nb} along the blades should be prepared for the rotors with different number of blades "b" for two or three values of blade mass constant " γ " for various values of the coefficient of advance μ , blade loading $\frac{C_T}{\sigma}$ and rotor angle of attack α.

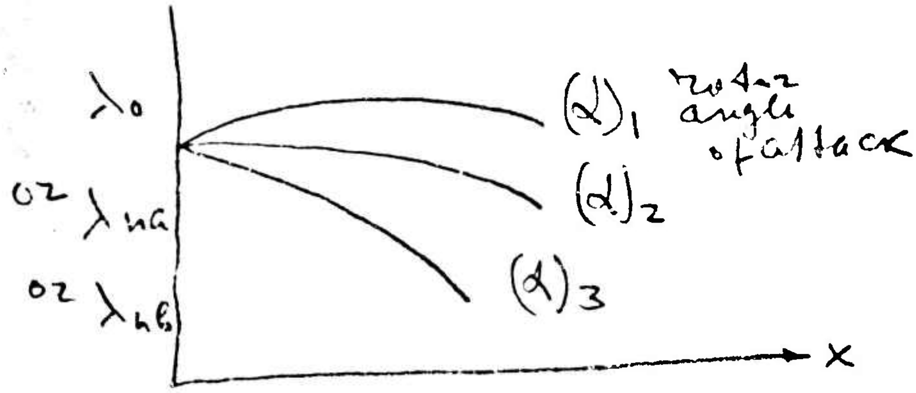
The sample chart is presented ~~below~~ on next page,

$\sigma =$

$\gamma =$

$\mu =$

$\frac{C_T}{\sigma} =$



The angle of attack " α " should be varied from near $\frac{-\pi}{2}$ which represents a steep climb to near $\frac{+\pi}{2}$ which is a steep glide.

It was suggested by Vertol Co., to plot the above coefficients against $\frac{C_T}{\sigma}$, $\frac{\lambda}{\mu}$, and $\frac{2\ \lambda}{\sigma}$ where $\lambda = \frac{V \sin\ \ \ - v\ \text{(Glauert)}}{\Omega R}$

The ratio $\frac{\lambda}{\mu}$ represents theoretical value of wake skew angle, the $\frac{2\ \lambda}{\sigma}$ ratio is proportional to the vertical space (pitch) of the helical vortex pattern beneath the rotor.

<u>Calculation of air load distribution on a stiff blade from charts.</u>

The lift airload distribution can be calculated from the expended expression of $\frac{dL}{dx}$. If only first harmonic flapping and two harmonics of induced velocity are considered, then from Ref. 1 (equation 4.83) the load contains three harmonics and they are

$$\frac{dL_0}{dx} = K\left[\theta\left(x^2+\frac{1}{2}\mu^2\right)+\bar{\lambda}x-\frac{1}{2}\mu\lambda_{\beta}\right] \qquad (20)$$

$$\frac{dL_{1a}}{dx} = K\left[b_1\left(x^2+\frac{\mu^2}{4}\right) - \beta_0 \mu x - \lambda_{1a} x - \frac{1}{2}\mu\lambda_{2b}\right] \tag{21}$$

$$\frac{dL_{1b}}{dx} = K\left[a_1\left(\frac{\mu^2}{4} - x^2\right) + 2\theta\mu x + \mu\left(\bar{\lambda} + \frac{1}{2}\lambda_{2a}\right) - x\lambda_{1b}\right] \tag{22}$$

$$\frac{dL_{2a}}{dx} = K\left[\frac{-\theta\mu^2}{2} + a_1\mu x - x\lambda_{2a} + \frac{\mu}{2}\lambda_{1b}\right] \tag{23}$$

$$\frac{dL_{2b}}{dx} = K\left[-\beta_0\frac{\mu^2}{2} + b_1\mu x - x\lambda_{2b} - \frac{\mu}{2}\lambda_{1a}\right] \tag{24}$$

$$\frac{dL_{3a}}{dx} = K\left(-\frac{1}{4}\mu^2 b_1 + \mu\frac{\lambda_{2b}}{2}\right) \tag{25}$$

$$\frac{dL_{3b}}{dx} = K\left(\frac{1}{4}\mu^2 a_1 - \mu\frac{\lambda_{2a}}{2}\right) \tag{26}$$

Where $K = 1/2\, \rho\, ac\Omega^2 R^3$ and $\bar{\lambda} = \frac{V \sin\alpha}{\Omega R} - \lambda_0$.
Parameter θ is computed from the average thrust of the rotor or

$$\theta = \left[\frac{2C_T}{a\sigma} - \int_0^1\left(\bar{\lambda}x - \frac{1}{2}\mu\lambda_{1b}\right)dx\right] \div \left(\frac{1}{3} + \frac{\mu^2}{2}\right) \tag{27}$$

The equations for β_0, a_1, and b_1 are obtained from the blade moment equation about the flapping hinge.

$$\beta_0 = \frac{\gamma}{2}\left[\frac{1}{4}\theta\left(1+\mu^2\right) + \int_0^1\left(\bar{\lambda}x^2 - \frac{1}{2}\mu\lambda_{1b}\right)\right] \tag{28}$$

$$b_1 = \left[\frac{\gamma}{3}\mu\beta_0 + 4\int_0^1\left(\lambda_{1a}x^2 + \frac{x}{2}\mu\lambda_{2b}\right)dx\right] \div \left(1+\frac{\mu^2}{2}\right) \tag{29}$$

$$a_1 = \left\{\frac{8}{3}\mu\theta + 4\int_0^1\left[\left(\bar{\lambda} + \frac{1}{2}\lambda_{2a}\right)\mu x - \lambda_{1b}x^2\right]dx\right\} \div \left(1-\frac{\mu^2}{2}\right) \tag{30}$$

It may be noted that, contrary to Glauert's theories, the lateral flapping is not only direct function of coning but also of sine components of induced velocity.

Once these characteristics are known, the lift air load distribution may be determined by equations 20-26. The torque producing air load distribution is determined by the standard methods or, for example

$$dQ = r\left(dL\left(\frac{U_P}{U_T}\right) + dD_0\right)$$

where dD_0 is the load due to profile drag.

It may be noted that all computations require knowledge of the rotor angle of attack "α". It may be found, that it is more convenient to treat "α" and "μ" both as two independent variables, their magnitude being limited by the power condition.

A sample calculation was presented by the writer and Dr. R.B. Gray in Ref. 2.

The harmonic coefficients were determined for a specific case using the data of Ref. 3.

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3. Meyer, John R. Jr. and Falabella, Gaetano Jr. : "An investigation of the experimental Aerodynamic loading on a model helicopter rotor blade, NACA TN2953, May 1953.